to a finite limit $e_m(\infty)$ as K increases. The dependence $e_m(K)$ obtained by determining the extremum of the function (5) is shown in Fig.2 by the dashed line. The relation is also non-monotonic and has a minimum. When the dimensionless reaction rate constant K tends to zero, the maximum value of the absolute magnitude of the charge on the particles $|e_m|$ obtained in the flow, also always tends to zero.

Figs.3 and 4 show the dependence of the distance x_m from the plane at which the charge on the particles reaches the maximum value in modulus, and of the dimensionless potential difference $\varphi_m = \varphi 8\pi e b n^\circ/(uE)$ on the parameter $1/\mathrm{Re}_E$ for various values of N, where $K = \infty$. We see that the quantity x_m depends weakly on the parameters N, Re_E and $x_m \sim 1$, and the presence of the aerosol particles has a considerable effect on the relation $\varphi_m(1/\mathrm{Re}_E)$ only when $N > 1$ and $\mathrm{Re}_E < 2$.

REFERENCES

1. SEDOV L.I., Mechanics of a Continuous Medium, Moscow, Nauka, 1976.
2. VATAZHIN A.B., GRABOVSKII V.I., LIKHTER V.A. and SHUL'GIN V.I., Electrogasdynamic Flows. Moscow, Nauka, 1983.
3. CHALMERS J.A., Atmospheric Electricity. Oxford, Pergamon Press, 1967.
4. ADACHI M., KOUSAKA Y. and OKUYAMA K., Unipolar and bipolar diffusion charging of ultrafine aerosol particles, J. Aerosol Sci. 16, 2, 1985.
5. SEDOV G.L. and CHERNYI L.T., Equations of electrohydrodynamics of weakly ionized aerosols with diffusive charging of the particles of the disperse phase. Izv. Akad. Nauk SSSR, MZhG, 1, 1986.
6. SMIRNOV B.M., Introduction to Plasma Physics. Moscow, Nauka, 1982.
7. COLE J.D., Perturbation Methods in Applied Mathematics. Waltham, Mass. Blaisdell Pub. Co. 1968.

Translated by L.K.

PMM U.S.S.R.,Vol.51,No.6,pp.805-807,1987
Printed in Great Britain

0021-8928/87 $10.00+0.00

THE PROBLEM OF THE FILLING OF A LIMITED VOLUME BY A VISCOUS HEAT-CONDUCTING GAS*

S.YA. BELOV

A system of differential equations, the solution of which describes the one-dimensional motion of a viscous heat-conducting ideal polytropic gas is investigated /1, 2/. It is proved that the problem of the filling of a limited volume by a gas is uniquely solvable. An existence theorem is established by the method of extending the solution that is local in time using global a priori estimates. A method of obtaining these estimates was described in /2/ for the equations of a viscous gas described in Lagrangian variables. The presence of penetrable walls means that the boundary conditions are non-uniform, and in mass Lagrangian variables the initial-boundary value problem is formulated in a region with curvilinear boundaries. This requires the development of a technique for proving the estimates. The correctness in time as a whole of the problem of the filling of a volume by a viscous gas has only been investigated previously for the more simple models, and for the system of equations of a heat-conducting gas in the case when the thermal conductivity depends in a special way on the temperature /3, 4/. Other formulations of the problem of the flows of a viscous gas in regions with penetrable boundaries were studied in /3-6/.

1. Formulation of the problem and fundamental results.

The one-dimensional motion of a viscous ideal polytropic gas in mass Lagrangian coordinates is described by the following system of equations /1, 2/:

$$\frac{\partial u}{\partial t} = \mu \frac{\partial}{\partial x}\left(\rho \frac{\partial u}{\partial x}\right) - \frac{\partial p}{\partial x}, \quad \frac{\partial \rho}{\partial t} + \rho^2 \frac{\partial u}{\partial x} = 0 \tag{1.1}$$

$$c_v \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(\varkappa \rho \frac{\partial \theta}{\partial x}\right) + \mu \rho \left(\frac{\partial u}{\partial x}\right)^2 - p \frac{\partial u}{\partial x}, \quad p = R\rho\theta$$

*Prikl.Matem.Mekhan.,51,6,1044-1046,1987

Here u, ρ, θ, p are the velocity, density, absolute temperature, and pressure respectively - the required characteristics of the medium, x is the mass Lagrangian variable, t is the time, $\mu, c_v, \varkappa$ are the viscosity, specific heat capacity, and thermal conductivity - positive constants, and R is the universal gas constant.

We will consider the motion of a viscous gas in a certain region of physical space, the left-hand boundary of which is fixed and penetrable (the gas constantly flows through it), and the right-hand boundary remains fixed, unpenetrable and thermally insulated. In a time $T, 0 < T < \infty$ this process can be described in Lagrangian variables by the solution of the system of Eqs. (1.1), which is defined in the region $Q_T = \{(x, t): 0 < t < T, \ s(t) < x < X\}$ and satisfies the following conditions ($X > 0$ is the initial mass of the gas):

$$u = u_0(x), \quad \rho = \rho_0(x), \quad \theta = \theta_0(x) \ \text{ for } \ t = 0, \quad x \in \bar{\Omega} \tag{1.2}$$

$$u = u_1(t), \quad \rho = \rho_1(t), \quad \theta = \theta_1(t) \ \text{ for } \ x = s(t), \quad t \in [0, T] \tag{1.3}$$

$$u = \frac{\partial \theta}{\partial x} = 0 \ \text{ for } \ x = X, \quad t \in [0, T] \tag{1.4}$$

$$s(t) = -\int_0^t \rho_1(\tau) u_1(\tau)\, d\tau, \quad \Omega = (0, X), \quad \Omega_t = (s(t), X)$$

In this case the following constraints are satisfied:

$$0 < m_0 \leqslant (\rho_0, \theta_0, \rho_1, u_1, \theta_1) \leqslant M_0 < \infty \tag{1.5}$$

(m_0 and M_0 are arbitrary positive constants).

We will formulate the main result, using the idea of a generalized solution and the symbols of functional spaces used in /2/.

Theorem 1. Suppose the given problems are such that

$$(u_0, \theta_0) \in C^{2+\alpha}(\Omega), \quad \rho_0 \in C^{1+\alpha}(\Omega), \quad (u_1, \rho_1, \theta_1) \in C^{1+\alpha/2}(0, T), \quad 0 < \alpha < 1$$

satisfy the conditions for matching of the first order of the initial and boundary functions and conditions (1.5). Then a unique classical solution of problem (1.1)-(1.4) exists, which has the properties

$$(u(x, t), \ \theta(x, t)) \in C^{2+\alpha, 1+\alpha/2}(Q_T), \quad \rho \in C^{1+\alpha}(Q_T), \quad \rho > 0, \ \theta > 0$$

If these problems belong to a wider class

$$(u_0, \rho_0, \theta_0) \in \dot{W}_2^1(\Omega), \quad (u_1, \rho_1, \theta_1) \in W_2^1(0, T)$$

satisfy the condition for matching of zeroth order, and condition (1.5) is satisfied, then a unique generalized solution exists such that the functions ρ and θ are strictly positive and bounded.

The existence of a unique classical solution over the whole time interval $[0, T]$ can be proved by extending the local solution using a priori estimates. After doing this, the generalized solution as a whole is obtained as the limit, defined in Q_T, of the classical solutions of the problems, the smooth initial and boundary data of which approximate the specified functions in the corresponding norms. The proof of the uniqueness of the generalized solution does not differ from that given in /2/ for a uniform initial-boundary value problem.

2. A priori estimates. The constants which depend only on the data of problem (1.1)-(1.4) and T, will be denoted by N (with a subscript).

Suppose the conditions of the first part of Theorem 1 are satisfied, and problem (1.1)-(1.4) has a classical solution, where $0 < \rho < \infty, 0 < \theta < \infty$. Over a small time interval this is guaranteed by the local existence theorem.

We will integrate the second equation of system (1.1), written in the form $(\rho^{-1})_t = u_x$, over the region $Q_t = \{(x, t): 0 < \tau < t, \ x \in \Omega_\tau\}$. We obtain the following relation:

$$\int_{\Omega_t} \rho^{-1}(x, t)\, dx = \int_{\Omega} \rho_0^{-1}(x)\, dx = N, \quad \forall t \in [0, T] \tag{2.1}$$

We substitute $u = w + u_2, \ \rho = v^{-1}$ into the equations of system (1.1), where

$$u_2(x, t) = u_1(t) N^{-1} \int_x^X \rho^{-1}(\xi, t)\, d\xi$$

and we multiply the first equation by w, the second by $R\theta_1 (1 - v^{-1})$, and the third by $(1 - \theta_1\theta^{-1})$, and we then integrate their sum over Q_t. After simple reduction, by estimating the right-hand side of the relation obtained with respect to the Cauchy inequality, and using Gronwall's lemma, we can obtain the estimate

$$\max_{0 \leqslant t \leqslant T} \int_{\Omega_t} [u^2(x, t) + (v(x, t) - \ln v(x, t) - 1)]\, dx \leqslant N_1 \tag{2.2}$$

which enables us to prove the following auxiliary assertion.

Lemma 1. The measured function $a(t)$ is defined in the interval $[0, T]$, such that $0 \leqslant a(t) \leqslant X$ and

$$N_2^{-1} \leqslant \rho(a(t), t) \leqslant N_3^{-1}, \quad \forall t \in [0, T] \tag{2.3}$$

where N_2 and N_3 are the roots of the equation $z - \ln z - 1 = N_1 X^{-1}$.

We will denote by $t = t^*(x)$, $s(T) \leqslant x \leqslant 0$ the function that is the inverse $x = s(t)$. Then $t^*(x) = 0$ if $0 \leqslant x \leqslant X$.

At any point $(x, t) \in \bar{Q}_0 = \{0 \leqslant x \leqslant X, 0 \leqslant t \leqslant T\}$, following /7/, we can obtain

$$\rho(x, t) = Y(x, t, t) B(x, t, t) J^{-1}(x, t, t) \tag{2.4}$$

where

$$Y(x, t, r) = \rho(a(r), t)\rho^{-1}(a(r), t^*(x)) \exp\left\{\mu^{-1} \int_{t^*(x)}^{t} p(a(r), \tau)\, d\tau\right\}$$

$$B(x, t, r) = \exp\left\{\mu^{-1} \int_{a(r)}^{x} [u(\xi, t^*(x)) - u(\xi, t)]\, d\xi\right\}$$

$$J(x, t, r) = \rho^{-1}(x, t^*(x)) + \mu^{-1} R \int_{t^*(x)}^{t} \theta(x, \tau) Y(x, \tau, r) B(x, \tau, r)\, d\tau$$

are functions defined in $G = \{(x, t, r): 0 \leqslant t \leqslant T, s(t) \leqslant x \leqslant X, 0 \leqslant r \leqslant t\}$.

At points of the curvilinear triangle $Q_1 = \{(x, t): 0 < t \leqslant T, s(t) \leqslant x < 0\}$ the following equations hold:

$$\rho(x, t) = Y(x, t, t) B(x, t, t) J^{-1}(x, t, t^*(x)) \tag{2.5}$$

$$\rho(x, t) = Y(x, t, t^*(x)) B(x, t, t^*(x)) J^{-1}(x, t, t^*(x)) \tag{2.6}$$

For an arbitrary function $f(x, t)$, continuous in $\bar{Q}_T$, we introduce the notation

$$m_f(t) = \min_{x \in \bar{\Omega}_t} f(x, t), \quad M_f(t) = \max_{x \in \bar{\Omega}_t} f(x, t)$$

By estimating the right-hand side of Eq. (2.4) from below and from above, and then the right-hand side of (2.5) from below and the right-hand side of (2.6) from above, we obtain the following assertion.

Lemma 2. For any $t \in [0, T]$ the following relations hold:

$$m_\rho(t) \geqslant N_4 + \left[1 + N_5 \int_0^t M_\theta(\tau)\, d\tau\right]^{-1}, \quad M_\rho(t) \leqslant N_6$$

After this all the estimates necessary to prove Theorem 1 are obtained using the scheme described in /2/.

Note. In a similar way we can investigate the problem when both boundaries of the region are penetrable and there is a flow of liquid through them.

REFERENCES

1. ROZHDESTVENSKII B.L. and YANENKO N.N., Systems of Quasilinear Equations and their Applications in Gas Dynamics, Nauka, Moscow, 1978.
2. ANTONTSEV S.N., KAZHIKOV A.V. and MONAKHOV V.N., Boundary Value Problems of the Mechanics of Non-Uniform Liquids, Nauka, Novosibirsk, 1983.
3. BELOV S.YA., The solvability "as a whole" of the problem of flow for Burgers' equation of a compressible fluid, in: Dynamics of a Continuous Medium, Inst. Hydrodynamics, Novosibirsk, 1981.
4. BELOV S.YA., The problem of flow for the system of equations of the one-dimensional motion of a viscous heat-conducting gas, in: Dynamics of a Continuous Medium, Inst. Hydrodynamics, Novisibirsk, 56, 1982.
5. KAZHIKOV A.V., Boundary value problems for Burgers' equations of a compressible fluid in regions with movable boundaries, in: Dynamics of a Continuous Medium, Inst. Hydrodynamics, Novosibirsk, 26, 1976.
6. BELOV S.YA., The problem of the filling of a vacuum by a viscous heat-conducting gas, in: Dynamics of a Continuous Medium, Novosibirsk, 59, 1983.
7. KAZHIKOV A.V. and SHELUKHIN V.V., The unique solvability "as a whole" in time of initial-boundary value problems for the one-dimensional equations of a viscous gas, PMM, 41, 2, 1977.

Translated by R.C.G.